DURATION AND ITS USE IN PUBLIC AGENCY INVESTMENT PORTFOLIOS

Doug Skarr
CDIAC Policy Research Unit

Duration is a term used by fixed-income investors, financial advisors, and investment advisors. At its most basic level, duration is a measurement of how long in years it takes for the price of a bond to be repaid by its internal cash flows. It is an important measure for investors to consider, as bonds with higher durations (given equal credit, inflation and reinvestment risk) may have greater price volatility than bonds with lower durations. It is an important tool in structuring and managing a fixed-income portfolio based on selected investment objectives. This article will discuss some basic concepts associated with duration math and its use in managing a bond portfolio.

Basic Bond Math and Risk Measurement

The price of a bond, or any fixed-income investment is determined by summing the cashflows discounted by a rate of return. The rate of return can change at any time period and will be reflected in the calculation of an investment's market price.

Figure 1 – Bond Pricing Formula

The price of a bond can be calculated using the formula found in Figure 1.

$$\text{Bond Price} = \sum_{t=1}^{n} \frac{\text{CPN}_t}{\{1+\text{YTM}\}} + \frac{\text{P}_n}{\{1+\text{YTM}\}}$$

Coupon Cash Flows | Principal Repayment

Definitions:

CPN	= coupon payment for period t
P	= principal payment
YTM	= yield to maturity
n	= number of compounding periods
t	= time period

In reviewing this basic formula, it can be seen that the sensitivity of a bond's value to changing interest rates depends on both the length of time to maturity and on the pattern of cashflows provided by the bond.

As shown, there are many variables associated with pricing a bond. Changes in each of these variables, taken separately and in combination, can have a significant effect on price.

What is Duration?

Duration is a calculation that brings all these factors together in one number, allowing a measurement of a bond's price sensitivity to changes in maturity and interest rates.

The common objective behind the different definitions of duration is to measure the price sensitivity or market risk of a fixed-income security to changes in its yield. Bonds of similar duration will have similar price movements for a given movement in interest rates. Duration comes in three forms:

Macaulay Duration. Developed in 1938 by Frederic Macaulay, this form of duration measures the number of years required to recover the true cost of a bond, considering the present value of all coupon and principal payments received in the future (which is why it is the only type of duration quoted in "years"). It assumes interest rates are continuously compounded.

Modified Duration. This measure expands or *modifies* Macaulay duration to measure the responsiveness of a bond's price to interest rate changes. It is defined as the percentage change in price for a 100 basis point change in interest rates. The formula assumes that the cash flows of the bond do not change as interest rates change, which is not the case for most callable bonds.

Effective Duration. Effective duration (sometimes called option-adjusted duration) further refines the modified duration calculation. Effective duration requires the use of a model for pricing bonds that adjusts the price of the bond to reflect changes in the value of the bond's "embedded options" (e.g., call options or a sinking fund schedule) based on the probability that the option will be exercised. All things being equal, as interest rates fall, bonds with embedded call options are exercised and the "in-the-money" bond is repaid. If interest rates rise, embedded options will not be exercised and the "out-of-the-money" bond will continue to maturity. Effective

duration will shorten and be closer to the call date for "in-the-money" bonds, while lengthening and being closer to the maturity date for "out-of-the-money" bonds.

Principles of Duration
Three factors that influence the duration calculation are ***coupon rate*** (which determines the size of the periodic cash-flow), ***interest rates*** (which determines the *present value* of the periodic cashflow), and ***maturity*** (which weights each cashflow) all contribute to the above duration measures. As a result, the two main principals of duration are:

As maturity increases, duration increases and the bond's price becomes more sensitive to interest rate changes.

- A decrease in maturity decreases duration and renders the bond less sensitive to changes in market yield. Therefore, duration varies directly with maturity.

As the bond coupon increases, its duration decreases and the bond's price becomes less sensitive to interest rate changes.

- Increases in coupon rates raise the present value of each periodic cashflow and therefore the market price. This higher market price lowers the duration.

Implications
- Duration allows bonds of different maturities and coupon rates to be directly compared.
- The higher the duration, the higher the risk of price changes as interest rates change.
- Constructing a bond portfolio based on weighted average duration provides the ability to determine value changes based on forecasted changes in interest rates.

Portfolio Duration
Treasury managers may manage interest rate risk by changing the duration of the portfolio. Portfolio duration strategies may include reducing duration by adding shorter maturities or higher coupon bonds. They may increase duration by extending the maturities or including lower-coupon bonds to the portfolio.

Each of these strategies can be employed based on the manager's propensity for active or passive investment management. If a treasury manager employs a passive management strategy, say targeting returns to a benchmark index, he or she may construct the portfolio to match the duration of the benchmark index. By contrast, an active strategy using benchmarks may include increasing the portfolio's duration to 105 percent of the benchmark during periods of falling rates, while reducing the duration to 95 percent of the benchmark during periods of rising rates.

Limitation to the Use of Duration in Bond Portfolios
Despite having important uses in bond investments, the basic duration measures (Macaulay and modified) have limitations:

- The interest rate sensitivity of a bond portfolio can only be estimated if there is a change in interest rates that leads to a parallel shift in the yield curve. In general, bonds of different maturities seldom experience the same change in rates (i.e., parallel shift in the yield curve). Thus, two bond portfolios that may have the same duration at the beginning of the investment horizon may end up being affected differently by interest rate changes one period later, depending on how the yield curve has moved.

- Neither basic duration formula can be used to estimate interest rate sensitivity of callable bonds because changes in interest rates may affect not only the prices of the bonds but the receipt of the cash flows as well. This is because the bonds may be called as a result of changes in interest rates. The use of *effective duration* modeling can significantly improve these limitations.

Conclusion
The objective of this article is to provide the reader a basic understanding of duration and its use in managing fixed income investment portfolios.

Duration is an important concept and tool available to all treasury managers. Treasury managers may use duration to develop investment strategies that maximize returns while maintaining appropriate risk levels in a changing interest rate environment. They may use duration as a tool for either passive or active management. For passive management, duration can be used in strategies that may insulate the value of the portfolio from changes in interest rates.

As with most financial management tools, duration does have certain limitations. A bond's price is dependent on many variables apart from the duration calculation and rarely correlates perfectly with the duration number.

Generally, interest rates do not move in parallel shifts and the yield curve is constantly changing. With this in mind, duration is not an absolute measure and can only be used to determine how the bond's price "should" react to changes in rates, as opposed to how it "will" react. Nevertheless, it is an important tool available to treasury managers in the administration of their fixed-income portfolios.

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